# Finite Element Modeling of Piezoelectric Nanobeams with Surface And Flexoelectricity Effects

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In this paper, a size-dependent Euler–Bernoulli beam model, which takes the flexoelectricity, piezoelectricity and dielectricity as well as the surface elasticity into consideration, is established. Theoretical solutions for the static bending deflection of thin beams under different loading (uniformly distributed and concentrated load) and boundary conditions (cantilevered, both ends simply supported, clamped-clamped), are established. Moreover, an iterative finite element algorithm is developed for the analysis thin flexoelectric beams under any load and boundary conditions. Our numerical results show the direct bulk flexoelectricity always play the role of stiffening the beams with various boundary conditions, while the residual surface stresses behave either stiffening or softening the nanobeams dependent on boundary conditions. The present investigation also demonstrates that the size-dependent effects of flexoelectricity on the bending rigidity and piezoelectricity.

The present investigations suggest the following conclusion:

(1) The direct bulk flexoelectricity plays the role of stiffening the beams with various boundary conditions, while the residual surface stresses behave either stiffening or softening the nanobeams dependent on boundary conditions. The positive and negative residual surface stresses soften and stiffen the cantilever beam, respectively. Other than the cantilever beam, the positive residual surface stresses stiffen the SS and CC beams and while the negative residual surface stresses soften the SS and CC beams. This phenomenon is attributed to different signs of the curvature of CF, SS and CC beams under the same loads. When the sign of the curvature is same as that of residual surface stress, the deflection is enhanced and vice versa.

(2) This paper discusses the size-dependent effect of flexoelectricity on the bending rigidity. It is concluded that the bending rigidity exhibits a strong dependence on the flexoelectric effect when the beam thickness is enough small. Compared with flexoelectric effect, the bending rigidity is weakly affected by surface elasticity and piezoelectric constant $e_{31}$. The size-dependent effect of flexoelectricity gradually decreases with the increases of the beam thickness and becomes negligible when the beam thickness increases to some extent.

(3) Our investigations also show the size-dependent enhancement of piezoelectricity by flexoelectric effect. It is observed that the normalized effective piezoelectric constant(i.e. the electromechanical response) increases with decreases in the beam thickness. Even for a non-piezoelectric material(i.e. $e_{31}$=0), the normalized effective piezoelectric constant $e^{'}$ of thin flexoelectric beam is still far larger than that of a pure piezoelectric beam as size shrinks. The simulations indicate the potential applications of flexoelectricity in the next-generation nano electronics and energy harvesters.

(4) Numerical simulations demonstrate the computational results of the present finite element are in good agreement with those of theory. The presented finite element formulation provides a robust analysis tool to understand the electromechanical coupling in nano-dielectrics and design various beams based nano-electromechanical systems under complex load and boundary conditions.